0604H2: Fracture

HW Exam-II: Topics Related to Fracture at Crack tips and Work of Fracture

Due Monday, Dec 13, 2021

1, 2 and 3

For each of the following cases: describe in words how the mechanism of fracture at the crack tip can be related to the work of fracture.

•Describe in words how the local parameters for fracture (they should be physically obvious) can be merged to calculate the work of fracture).

•Explain how the units for above parameters can be combined to obtain the work of fracture which has units of J m^{-2}.

1. Case I: ideal or brittle fracture

2. Case II: fracture by small scale yielding at the crack tip which leads to local plastic tearing.

3. Case III: fracture by the stretching and breaking of fibrils at the crack tip in polymers.

4. Type: Case I (Part I)

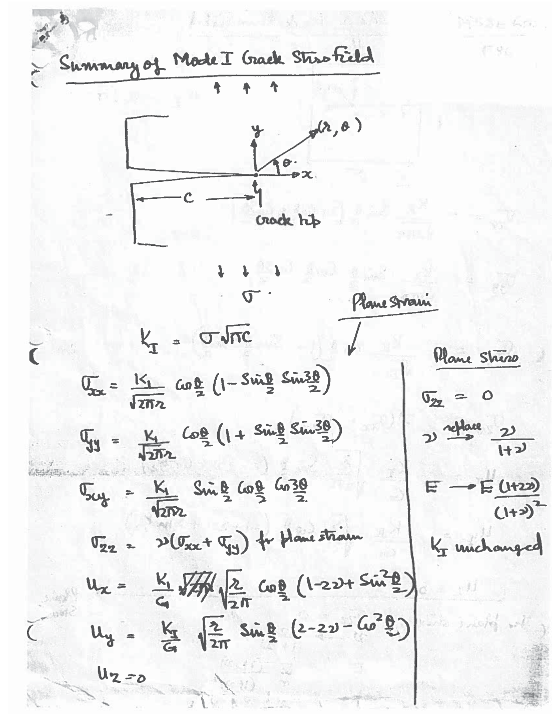
The sine-wave model for bond rupture, discussed in today's class, were total separation was assumed to be achieved with a stretch displacement equal to one half of the interatomic spacing, that is $0.5\Omega^{1/3}$, that is, half the sine wave stretches from 0 to $0.5\Omega^{1/3}$.

With the above assumption, **please derive** that the elastic strain at maximum stress is given by

$\varepsilon_f = \dfrac{1}{2\pi}$, and the maximum stress (at fracture) = $\varepsilon_f E$ where E is the elastic modulus.

Show that the above result when substituted into the equations for stress near the crack tip (in terms of the applied stress intensity factor - the equations are given on the following page), leads to the following expression for K_{IC}

$$\sigma_{yy}^{*}(\theta = 0, r = \Omega^{1/3}) = \frac{K_{IC}}{\sqrt{2\pi\Omega^{1/3}}} = \varepsilon_f E \qquad (1)$$


Summary of Mode I Crack Stress Field

$$K_I = \sigma\sqrt{\pi c}$$

Plane strain

$$\sigma_{xx} = \frac{K_1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{yy} = \frac{K_1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{xy} = \frac{K_1}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) \text{ for plane strain}$$

$$u_x = \frac{K_1}{G}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(1 - 2\nu + \sin^2\frac{\theta}{2}\right)$$

$$u_y = \frac{K_I}{G}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(2 - 2\nu - \cos^2\frac{\theta}{2}\right)$$

$$u_z = 0$$

Plane stress

$$\sigma_{zz} = 0$$

$$\nu \xrightarrow{\text{replace}} \frac{\nu}{1+\nu}$$

$$E \rightarrow E\frac{(1+2\nu)}{(1+\nu)^2}$$

K_I unchanged

5. Type: Case I (Part II)

In this problem you are asked to show whether or not the following universal equation in linear elastic fracture mechanics

$$\frac{K_{IC}^2}{E} = 2\gamma_S$$

is obeyed in the atomistic calculation of the surface energy from the point of view of work done to break a bond.

Note that:

•the work done is equal to the area under the half sine wave

•this is the work done per one bond which occupies and area of $\Omega^{2/3}$ of the surface. Show the units of this work are in Joules

•you must multiply by the number of bonds per unit area to find the expression of $2\gamma_S$ which has units of J m^{-2}.

Compare your result with Eq. (1). There will probably be a discrepancy.

•Check and recheck your algebra to be absolutely sure that you have the correct analysis.

•If there is still a difference - please give your reasons. Which assumption is likely to be incorrect, and how can you demonstrate your conclusion.